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Modelling the orthic tetrakaidecahedron

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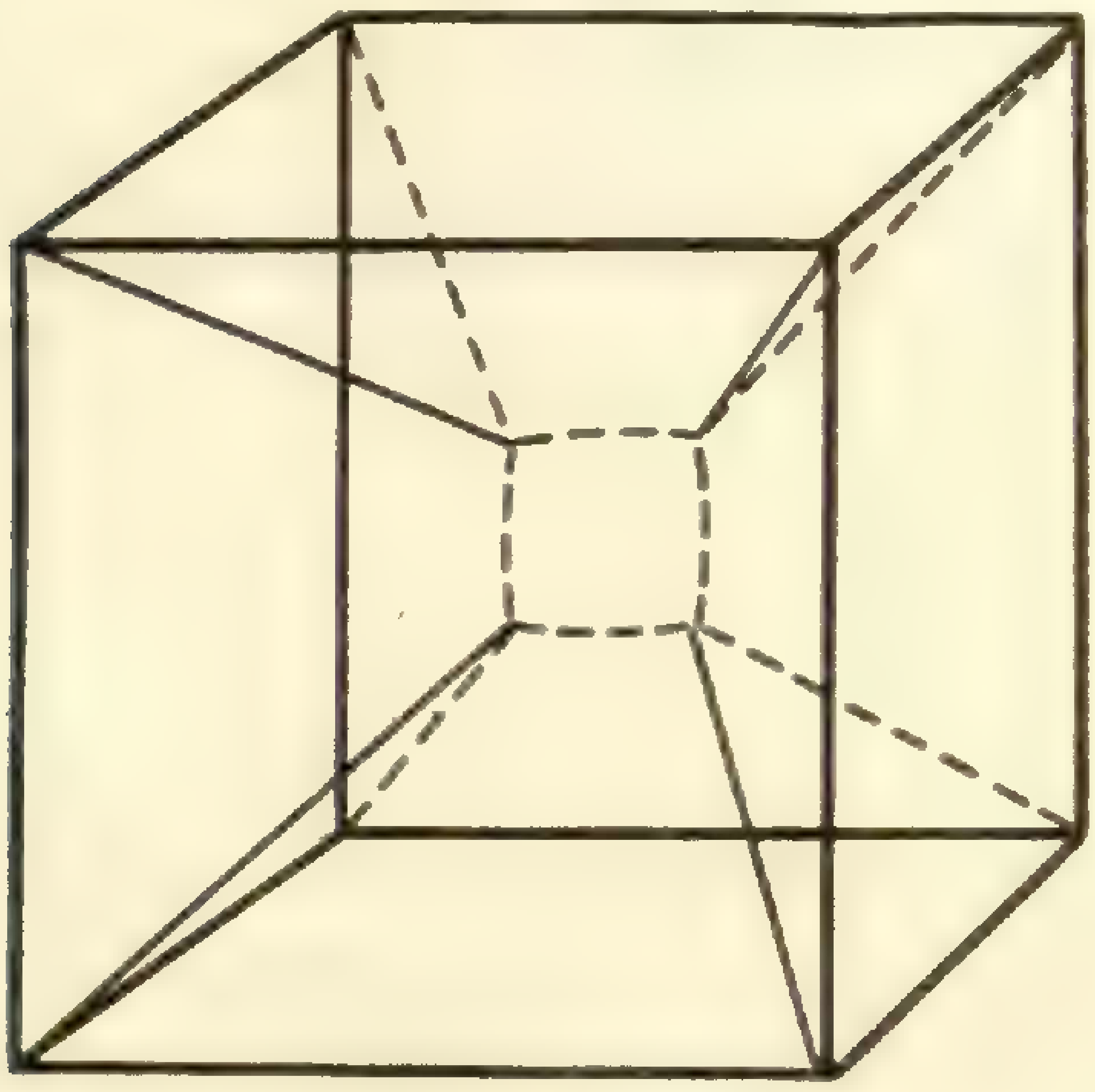
One of the most fascinating pages in the history of science is that which relates the discovery of the planet Neptune. After long and involved mathematical computations, Adams in England and Leverrier in France explained the movement of Uranus by the existence of a previously unknown planet, and they determined its approximate position. On the basis of Leverrier's calculations, Galle, in Berlin, was able almost immediately to locate the new planet.

Less spectacular, perhaps, less heralded, but no less scientific has been the investigation of cell shapes in the organic world by Kelvin and Lewis. As far back as 1887 Kelvin published an essay "On the division of space with minimum partitional area," in which he described a fourteen-sided figure that he called a tetrakaidecahedron. A similar figure had been known to the crystallographers even before Kelvin's publication. In contrast to the search for Neptune, which went on with almost feverish haste after the supposed orbit had been approximately determined, Kelvin's suggestion lay fallow for thirty-six years; it was only in 1923 that Lewis showed that cells of elder pith tend to be fourteen-sided, and at times show an alternation of hexagonal and square faces suggestive of Kelvin's figure. Lewis (1925, 1928) has since extended his observations, and gives data showing the primarily tetrakaidecahedral form of such diverse tissues as the stellate cells of *Juncus*, cells of human adipose and oral epithelial tissues, and cork cells, while Hein (1930 a, b) comes to similar conclusions in studying sclerotial tissue of the fungi. From a mathematical standpoint the orthic tetrakaidecahedron has been considered in a previous publication (Matzke 1927).

Kelvin apparently arrived at his tetrakaidecahedron from studying the cubic skeleton frame of Plateau (1873). This is reproduced in figure 1—the frame being shown by the heavy lines.

If this frame is dipped into a soap solution and then withdrawn, the soap films form a quadrilateral face in the center, and each of the four sides of the quadrilateral also forms a side of a half-hexagon. This relationship is similar to a quadrilateral face of Kelvin's tetrakaidecahedron, which is surrounded by four hexagonal faces (as shown in figure 3). The limits of the three half-hexagons of figure 1 are represented by dotted lines in figure 3. The region enclosed by the dotted lines in figure 3 thus corresponds to the dotted portion of figure 1. Kelvin objected to the rhombic dodecahedron as a form for the partitioning of space



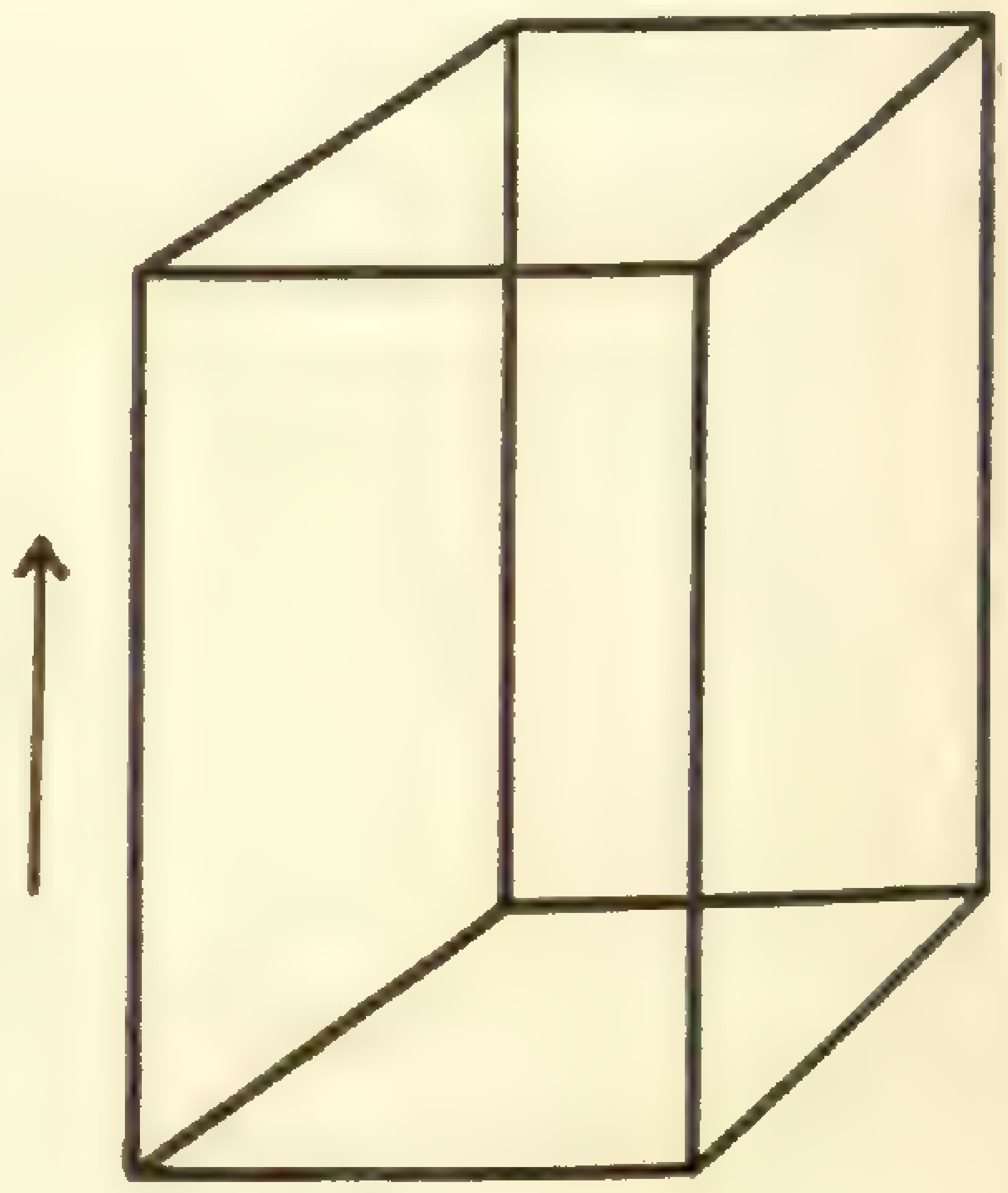


FIGURE 1. Cubic skeleton frame of Plateau with soap films, a quadrilateral face and four half-hexagonal faces of Figure 3 shown by dotted lines.

FIGURE 2. Skeleton frame of a rectangular prism, to show the slipping of the soap films; it should be removed from the soap solution as indicated by the arrow.

because of its tetrahedral angles which, as he said, are essentially unstable. By blowing gently on the edge of the quadrilateral face in figure 1, that is by blowing parallel with its surface, that face will decrease in size, one side of the half-hexagonal faces (which are also quadrilaterals) thus becoming smaller and smaller. However, as Kelvin pointed out, this quadrilateral face cannot be made to disappear completely, for just as it approaches the vanishing point there is a readjustment of the soap films, and a new quadrilateral face is formed, its surface being perpendicular to the surface of the first.

A similar slipping or readjustment of the soap films can be demonstrated by using instead of the cubic skeleton frame, the frame of a rectangular prism (figure 2). If this is dipped into a thick soap solution, and then removed, the direction of removal being parallel to the long axis of the frame (as indicated by the arrow in figure 2), there will be a quadrilateral face in the center which grows smaller and smaller; just as it vanishes a new face appears at right angles to the original one. This is easily demonstrable, the films slipping slowly, if the solution is dense enough. As known to Plateau and others, a soap solution to which glycerine has been added gives satisfactory results.

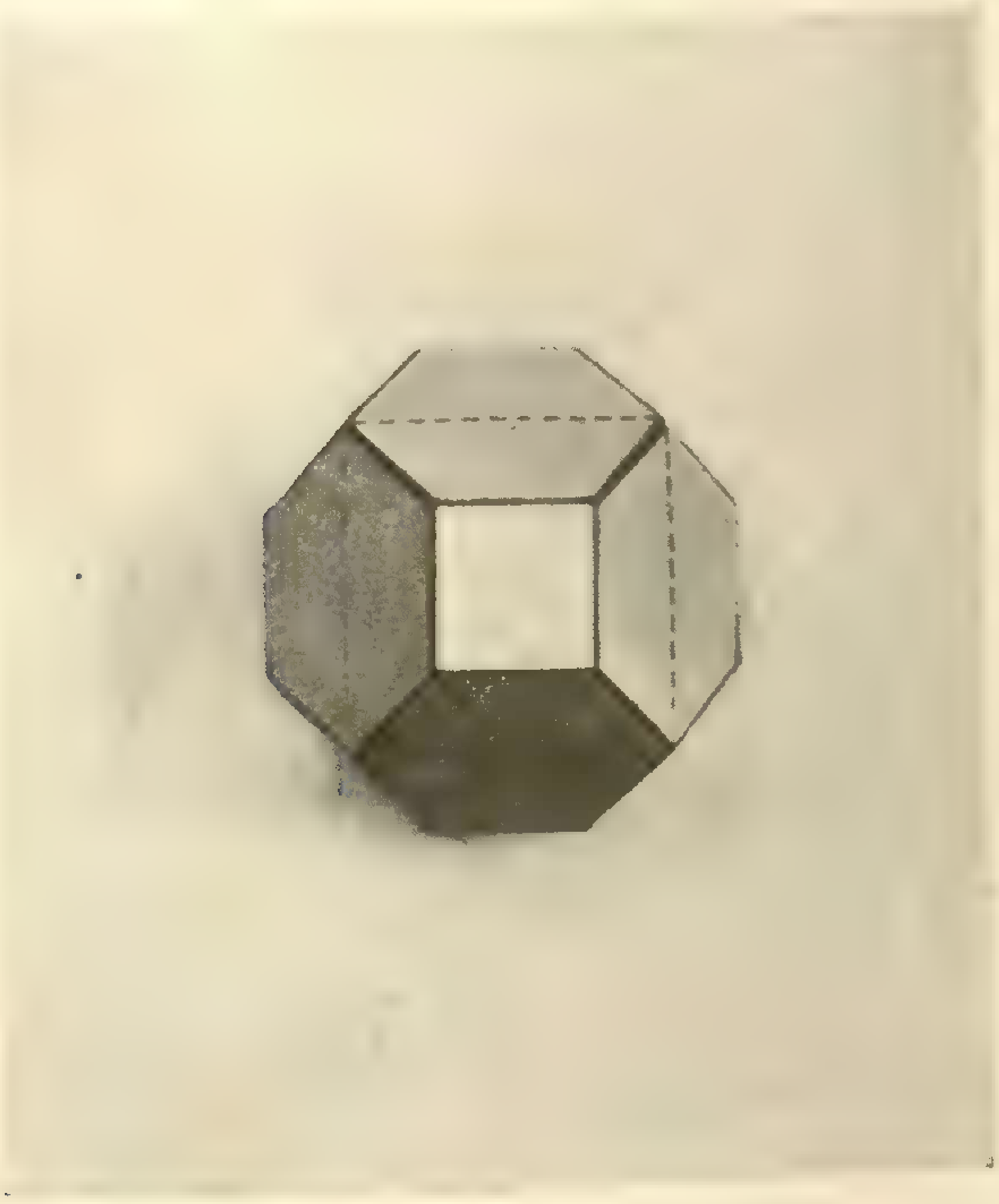


FIGURE 3. Photograph of an orthic tetrakaidecahedron, showing a square face bordered by four hexagonal faces. The four half-hexagonal faces of Figure 1 are limited by the dotted lines.

The orthic tetrakaidecahedron may be modelled in various ways. Kelvin (1894) suggested soldering together thirty-six pieces of wire. Figure 4 shows a layout for a paper model; this, printed on stiff paper, can be cut out, and folded on the heavy lines and glued by means of the flaps. It can be enlarged by means of the photostat. It is obvious that this is by no means the only arrangement possible for a paper model, but it is the most convenient. By using thin sheet metal instead of paper, and making the model in two parts, as indicated in figure 5, a mold can be made; then by soldering the edges of each half of this mold, and temporarily fastening the two halves together, a form in which wax models can be poured is obtained.

The study of cell shapes in three dimensions is still in its infancy; only a few tissues have been investigated—largely, without doubt, because of the difficulty of making accurate determinations. In addition to the form of the mature cells, the Strasburgers of the future will unfold, step by step, the changes that the cells undergo, from the time of their origin, usually at the growing apex, to the time that they are fully differentiated. Is it too sanguine to hope that this process will not merely be traced, but understood in terms of the mechanics by which it is underlain?

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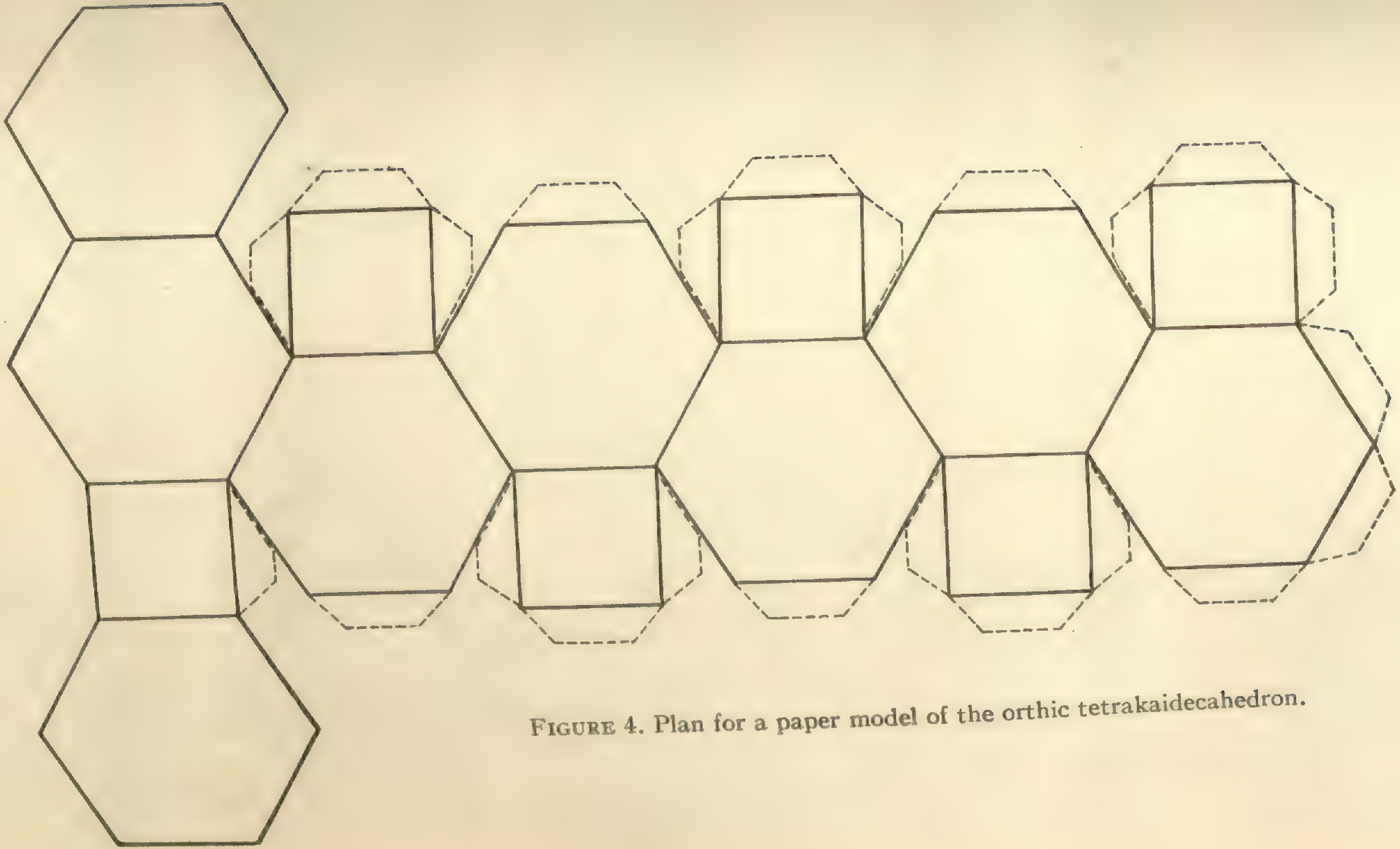

FIGURE 4. Plan for a paper model of the orthic tetrakaidecahedron.

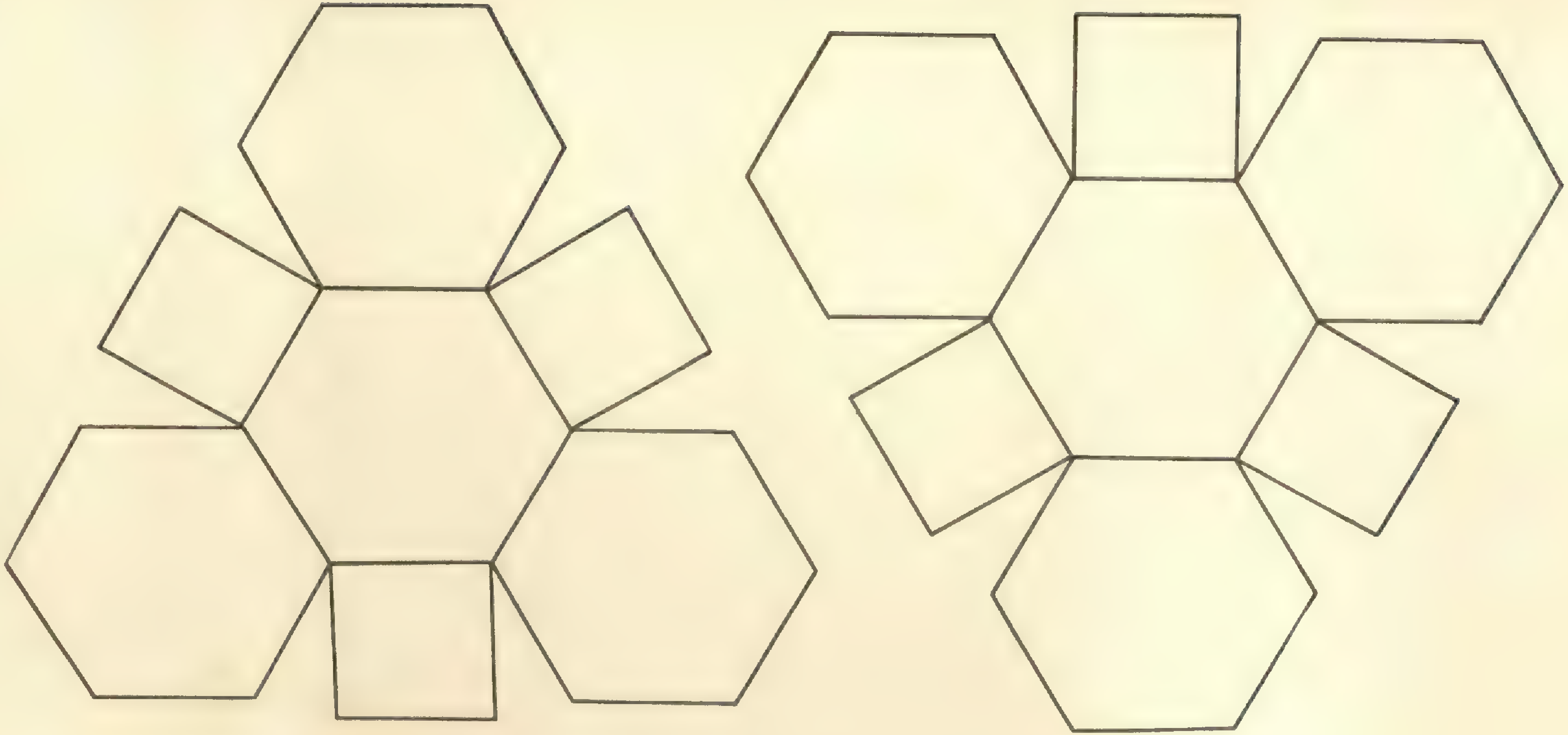

FIGURE 5. Plan for a metal mold of the orthic tetrakaidecahedron.